Exam. Code : 209001
Subject Code : 4763

M.Sc. Physics 1st Semester
MATHEMATICAL PHYSICS
Paper—PHY-402

Time Allowed—Three Hours] [Maximum Marks—100

Note :— Candidates are to attempt **FIVE** questions, **ONE** from each Section. **Fifth** question may be attempted from any Section. All questions carry equal marks.

SECTION—A

1. (a) State and prove convolution theorem for Fourier Transform. What does physically it represent ? 10

(b) Find the Fourier Transform of :

$$f(x)=\begin{cases} 1-|x| & \text{if } |x|<1 \\ 0 & \text{if } |x|>1 \end{cases}$$

Hence prove that :

$$\int_0^\infty \left(\frac{\sin t}{t}\right)^4 dt = \frac{\pi}{3}\cdot$$ 10

2. (a) Construct a scalar from the tensor A^{ij}_{kl}. 5

(b) If $ds^2 = g_{ij}dx^i dx^j$ is invariant, show that g_{ij} is a symmetric covariant tensor of second rank. 5

(c) Using the operator formalism of orbital angular momentum, prove that :

$$\vec{L} \times \vec{L} = \iota\vec{L}.$$ 10

SECTION—B

3. The interaction of two particles is described by a potential $V(x) = A\dfrac{e^{-ax}}{x}$, where A is negative constant. Solve the following resultant Schrodinger equation :

$$\frac{\hbar^2}{2\mu}\frac{d^2y}{dx^2} + (E - V)y = 0.$$ 20

4. (a) Write down an expression for the generating function of Bessel function, J_n. Use it to prove that $\dfrac{dJ_n}{dx} = \dfrac{1}{2}(J_{n-1} + J_{n+1})$ for the case of an integer n. 10

(b) Define Gamma function, $\Gamma(n)$, show that :

$$\Gamma\left(\frac{1}{2} - n\right)\Gamma\left(\frac{1}{2} + n\right) = (-1)^n \pi.$$ 10

SECTION—C

5. (a) Find the analytic function $w(z) = u(x, y) + \iota v(x, y)$, (i) if $u(x, y) = x^3 - 3xy^2$, (ii) if $v(x, y) = e^{-y} \sin x$. 10

(b) State and prove Cauchy residue theorem. Hence evaluate the integral $\int_0^{2\pi} \frac{d\theta}{3 - 2\cos\theta + \sin\theta}$. 10

6. (a) If f(z) is a real function of the complex variable $z = x + \iota y$ and the Laurent expansion about the origin, $f(z) = \Sigma a_n z^n$, has $a_n = 0$ for $n < -N$, show that all the coefficients of a_n are real. 10

(b) Show that $\int_{z_0}^{z} Z^n \, dz = \frac{z^{n+1} - z_0^{n+1}}{n+1}$ for all n, except $n = -1$. What physically $n = -1$ represent ? 10

SECTION—D

7. (a) Define permutation group and give one example. Discuss their importance in quantum mechanics of identical particles. 10

(b) Define the Unitary matrices and show that unitary matrices of order n form a group under matrix multiplication. 10

8. (a) Explain with at least two examples what are the differences between isomorphism and homomorphism. 10

(b) Define SO(2) and SO(3) are the rotational groups. What are the basic differences between these groups ? 10